Engineering Maths First Aid Kit

7.6

Multiplication and division in polar form

Introduction

When two complex numbers are given in polar form it is particularly simple to multiply and divide them. This is an advantage of using the polar form.

1. Multiplication and division of complex numbers in polar form.

if $z_1 = r_1\angle\theta_1$ and $z_2 = r_2\angle\theta_2$ then

$$z_1z_2 = r_1r_2\angle(\theta_1+\theta_2), \qquad \frac{z_1}{z_2} = \frac{r_1}{r_2}\angle(\theta_1-\theta_2)$$

Note that to multiply the two numbers we multiply their moduli and add their arguments.

To divide, we divide their moduli and subtract their arguments.

Example

If $z_1 = 5\angle(\pi/6)$, and $z_2 = 4\angle(-\pi/4)$ find a) z_1z_2, b) $\dfrac{z_1}{z_2}$, c) $\dfrac{z_2}{z_1}$

Solution

a) To multiply the two complex numbers we multiply their moduli and add their arguments. Therefore

$$z_1z_2 = 20\angle\left(\frac{\pi}{6}+\left(-\frac{\pi}{4}\right)\right) = 20\angle\left(-\frac{\pi}{12}\right)$$

b) To divide the two complex numbers we divide their moduli and subtract their arguments.

$$\frac{z_1}{z_2} = \frac{5}{4}\angle\left(\frac{\pi}{6}-\left(-\frac{\pi}{4}\right)\right) = \frac{5}{4}\angle\frac{5\pi}{12}$$

c)

$$\frac{z_2}{z_1} = \frac{4}{5}\angle\left(-\frac{\pi}{4}-\frac{\pi}{6}\right) = \frac{4}{5}\angle\left(-\frac{5\pi}{12}\right)$$

Exercises

1. If $z_1 = 7\angle\frac{\pi}{3}$ and $z_2 = 6\angle\frac{\pi}{2}$ find a) z_1z_2, b) $\frac{z_1}{z_2}$, c) $\frac{z_2}{z_1}$, d) z_1^2, e) z_2^3.

Answers

1. a) $42\angle\frac{5\pi}{6}$, b) $\frac{7}{6}\angle-\frac{\pi}{6}$, c) $\frac{6}{7}\angle\frac{\pi}{6}$, d) $49\angle\frac{2\pi}{3}$, e) $216\angle\frac{3\pi}{2}$.